Mathematical Analysis - List 15

1. Let $f(x,y) = \dfrac{x^2}{x^2+y}$, $x^2+y \neq 0$. Examine what happens when $(x,y) \to (0,0)$ along the curve $y = kx^2$ for different values of k and show that f does not have a limit at $(0,0)$.

2. Let $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{for} \quad (x,y) \neq (0,0) \\ 0 & \text{for} \quad (x,y) = (0,0) \end{cases}$.

a) Show that $f(0,y)$ and $f(x,0)$ are each continuous functions of one variable.
b) Show that rays emanating from the origin are contained in contours of f.
c) Is f continuous at $(0,0)$?

3. Let $f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{for} \quad (x,y) \neq (0,0) \\ 0 & \text{for} \quad (x,y) = (0,0) \end{cases}$. Compute $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

4. Explain why the following function is not continuous along the line $y = 0$.

$$f(x,y) = \begin{cases} 1-x, & y \geqslant 0, \\ -2, & y < 0. \end{cases}$$

5. Determine whether there is a value for c making the function continuous everywhere. If so, find it. If not, explain why not.

a) $f(x,y) = \begin{cases} c+y, & x \leqslant 3, \\ 5-y, & x > 3, \end{cases}$

b) $f(x,y) = \begin{cases} c+y, & x \leqslant 3, \\ 5-x, & x > 3. \end{cases}$

6. Determine whether there is a value for a making the function continuous at $(0,0)$. If so, find it. If not, explain why not.

a) $f(x,y) = \begin{cases} \dfrac{\sin xy}{y} & \text{when} \quad y \neq 0, \\ a & \text{when} \quad y = 0, \end{cases}$

b) $f(x,y) = \begin{cases} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} & \text{for} \quad (x,y) \neq (0,0) \\ a & \text{for} \quad (x,y) = (0,0). \end{cases}$